Research Report

Tolerable Variation in Item Parameter Estimates for Linear and Adaptive Computer-based Testing

Saba Rizavi

Walter D. Way

Tim Davey

Erin Herbert

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Saba Rizavi, Walter D. Way, Tim Davey, and Erin Herbert

ETS, Princeton, NJ

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Abstract

Item parameter estimates vary for a variety of reasons, including estimation error, characteristics of the examinee samples, and context effects (e.g., item location effects, section location effects, etc.). Although we expect variation based on theory, there is reason to believe that observed variation in item parameter estimates exceeds what theory would predict. This study examined both items that were administered linearly in a fixed order each time that they were used and items that had appeared in different adaptive testing item pools. The study looked at both the magnitude of variation in the item parameter estimates and the impact of this variation in the estimation of test-taker scores. The results showed that the linearly administered items exhibited remarkably small variation in parameter estimates over repeated calibrations. Similar findings with adaptively administered items in another high stakes testing program were also found when initial adaptively based item parameter estimates were compared with estimates from repeated use. The results of this study also indicated that context effects played a more significant role in adaptive item parameters when the comparisons were made to the parameters that were initially obtained from linear paper-and-pencil testing.

Key words: Item parameter estimates, computer adaptive testing, IRT, scoring

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Computer-based testing, and adaptive testing in particular, typically depends upon item response theory (IRT). The advantages of IRT are well-known through the testing literature and have fueled the transition of computerized adaptive testing (CAT) from a research interest to a widely used practical application. However, the introduction of computer-based testing in high volume, high stakes settings has presented new challenges to testing practitioners. In most computer-based testing programs, it is necessary to administer items repeatedly over time. This continuous item exposure raises security concerns that were not fully appreciated by researchers when the theory and practice of CAT were first developed.

In most CAT programs, steps are taken to protect the integrity of item pools through strategies such as item exposure control, pool rotation, and accelerated item development (Way, 1998). Despite such efforts, maintaining CAT programs remains difficult because adaptive algorithms tend to select the most highly discriminating items. Efforts to increase item development bring increased costs and diminishing returns. As these items become exposed and are retired from use, developing sufficient replacement items of the same quality is very difficult: Three or four items may need to be written to find a suitable replacement. Furthermore, the lag time between the initial writing of the items and use of the items in an operational CAT pool is usually significant, as items must be pretested, calibrated, and evaluated before they may be used operationally.

Recently, researchers at ETS have begun exploring an approach to adaptive testing that could address some of the challenges of item exposure and pool maintenance (Bejar et al., 2002). Bejar (1991) referred to this approach as generative testing. More recently it has been called *item modeling*. The essence of item modeling is to create items from explicit and principled rules. The approach has roots in computer-assisted instruction and domain-referenced testing (Hively, 1974). The obvious vehicle for item modeling is the computer, and successful applications of automated item generation have been reported by a number of researchers (Embretson, 1999; Irvine, Dunn, & Anderson, 1990; Irvine & Kyllonen, 2001).

Although the capability to develop item models and generate items automatically is more easily established for some item types than for others, the potential utility of automated item generation for supporting computer-based testing is obvious. An effective item model provides the basis for a limitless number of items, each of which is assumed to share the same content and statistical characteristics. In CAT, the adaptive algorithm could choose an item model based on

the common psychometric characteristics, and the actual instance of the item would be generated at the time of delivery. Such an approach was referred to as *on-the-fly* adaptive testing by Bejar et al. (2002). They carried out a feasibility study of a CAT application where item models were utilized and concluded that the adaptive generative model they employed was both technically feasible and cost effective.

From a traditional IRT perspective, the use of item models with adaptive testing seems far-fetched. In fact, much of the IRT literature in recent years has centered on item parameter estimation and parameter recovery, the idea being that successful applications of IRT depend upon well-estimated parameters. The notion that one could use a single set of IRT estimates to characterize all of the items generated from a particular model directly contradicts the goal of obtaining accurate item parameter estimates. However, such a perspective does not account for the variation that may occur in student scores due to a variety of effects that influence how test items are responded to in the real world. These include context effects, item position effects, instructional effects, variable sample sizes, and other sources of item parameter drift that are typically not formally recognized or controlled for in the context of CAT.

Several researchers have documented the existence and influence of such item level effects. Sireci (1991) looked at the effect of sample sizes on the stability of IRT item parameter estimates. Kingston and Dorans (1984) described such effects in equating the paper-and-pencil GRE. Leary and Dorans (1985) and Brennan (1992) reviewed literature related to context effects and provided guidelines on how such effects might be minimized. Zwick (1991) described a case study of how context effects created an anomaly in the Reading test scores on the National Assessment of Educational Progress (NAEP). Divgi (1986) documented changes in item parameter estimates in an early application of the Armed Services Vocational Aptitude Battery (CAT-ASVAB). Several researchers have investigated causes of item parameter drift in testing programs that utilized IRT in test construction and equating over time (Eignor & Stocking, 1986; Kolen & Harris, 1990; Sykes & Fitzpatrick, 1992; Way, Carey, & Golub-Smith, 1992).

In considering the viability of item models for CAT, we recognize that variation within models introduces a source of errors that is not present in traditional CAT. However, the repeated use of the same items across different CAT pools also introduces a source of errors that is tolerated but not accounted for. The purpose of this study was to investigate and to quantify the error that is currently tolerated in item parameter estimates for different sets of items used in

computer-based testing. The study examined items that were administered repeatedly to different examinee samples over time. We examined items that, each time they were used, were administered linearly in a fixed order and also items that had appeared in different adaptive testing item pools. We examined both the magnitude of variation in the item parameter estimates and the impact of this variation on the test takers' scaled or reported scores.

Case Study 1: Linear Administration of Items

Data

In order to carry out the investigation of the stability of parameter estimates in linearly administered tests, two sets of items from a high stakes admissions test were chosen. The first set was composed of 28 items from the Quantitative (QNT) measure, while the second set consisted of 30 items from the Verbal (VBL) measure of the same test. Since ability distributions for the Quantitative measure are known to change more rapidly than the other measures, a greater variation in the parameter estimates was expected for that measure.

The items contained in the two sets come from actual test administrations in which these items were used as anchors to place parameter estimates on the base scale for other items in the linearly administered pretest sections. In every online pretest calibration for these CAT programs, anchor items are administered as similarly as possible to pretest items. The composition of the anchor set mirrors the pretests in terms of psychometric and content characteristics; the number of items in the pretest and anchor set is the same. The Verbal and Quantitative anchor items evaluated in the study were used over a 2-year period and were calibrated for each administration of the corresponding pretest measure. Thus nine repeated calibrations were available for each anchor item. The average item parameter estimates for both Quantitative and Verbal measures are presented in Table 1.

The calibration samples were randomly obtained by spiraling the pretest forms across examinees. The sample sizes used to calibrate each item varied from 627 to 2,305 for the Quantitative measure and 830 to 2,284 for the Verbal measure. The details of sample sizes used for each calibration are presented in Table A1. The perfect response patterns were excluded from each of the response sets; the resulting sample sizes are presented in the last column of that table.

Table 1

Average Item Parameter Estimates (*a, b*)

Calibration	*a*-parameter		*b*-parameter	
	Mean	St. dev	Mean	St. dev
QNT				
1	0.842	0.342	–0.020	1.163
2	0.852	0.348	–0.008	1.145
3	0.826	0.340	–0.028	1.166
4	0.764	0.365	–0.042	1.341
5	0.764	0.297	–0.030	1.256
6	0.779	0.272	0.016	1.208
7	0.818	0.339	–0.047	1.235
8	0.755	0.335	–0.182	1.305
9	0.775	0.299	–0.036	1.175
VBL				
1	1.003	0.292	–0.090	1.150
2	0.983	0.290	–0.034	1.205
3	0.954	0.279	–0.065	1.222
4	0.967	0.270	0.035	1.229
5	1.024	0.291	0.045	1.124
6	1.129	0.313	0.046	1.059
7	0.970	0.278	–0.010	1.114
8	1.020	0.301	0.010	1.107
9	0.954	0.275	–0.061	1.186

Parameter Estimation Methodology

The item parameter estimates were obtained using the software LOGIST (Wingersky, Patrick, & Lord, 1988). LOGIST uses the joint maximum likelihood estimation methodology to estimate item parameters, keeping the ability parameters fixed, while formulating item parameter estimates. The ability parameters in this case were the actual ability estimates obtained on the operational section of the test. The estimates on the linearly administered items were then subjected to scaling using the test characteristic curve methodology proposed by Stocking and Lord (1983). In this study, the stability of estimates on both sets of anchor items was investigated after the scaling was carried out.

Analyses

In order to look at the general trends in the variation of individual parameter estimates the *a*, *b*, and *c* parameters were plotted for each item across calibrations. The purpose of this analysis was simply to get an idea of any directional change that could occur in some items over time. In order to look at the effect of parameter estimate variation on the probability of getting an item correct, the item characteristic curves were examined for each item across nine calibrations for both measures. The weighted Root Mean Squared Errors (RMSE) were then computed between the item characteristic curves for the various calibrations in relation to the first calibration. In other words the first calibration was chosen as a point of reference for all comparisons in this case. The RMSE in this case is defined as

$$RMSE_{ic} = \sqrt{\sum_{j=1}^{n} w_j (P_{ic}(\theta_j) - P_{i1}(\theta_j))^2}, \tag{1}$$

where $P_{ic}(\theta_j)$ is the probability of getting an item (*i*) correct in a calibration (*c*) at an ability level θ_j. The weight w_j is the proportion of examinees out of the total number of examinees and *n* is the number of ability levels. The ability levels (and the corresponding weights) were derived from the reference paper-and-pencil (P & P) base form ability distribution for this particular program on the number-right scale. The number-right score levels ranged from 10 to 59 for Quantitative and 15 to 75 for Verbal resulting in 11 and 13 ability levels for the two measures respectively. The levels were then converted on to the theta metric as listed in Table 2. These ability levels ranged from –3.839 to 3.546 for Quantitative and –5.855 to 4.881 for Verbal.

This index was used in similar research performed at ETS where the item characteristic curves (ICCs) obtained on different calibrations were compared (Guo, Stone, & Cruz, 2001; Rizavi & Guo, 2002). The RMSEs were then plotted for each item across calibrations to capture variation for items.

Table 2

Ability Levels and Corresponding Weights for Quantitative and Verbal Measures

	Quantitative		Verbal	
Level	Ability	Weight	Ability	Weight
1	–3.839	0.001	–5.855	0.000
2	–2.184	0.029	–3.355	0.006
3	–1.381	0.100	–2.337	0.019
4	–0.812	0.158	–1.635	0.049
5	–0.348	0.172	–1.074	0.111
6	0.053	0.155	–0.585	0.175
7	0.427	0.125	–0.127	0.195
8	0.807	0.106	0.329	0.163
9	1.242	0.094	0.800	0.130
10	1.882	0.055	1.298	0.084
11	3.546	0.003	1.856	0.051
12			2.608	0.017
13			4.881	0.000

Another interesting way to look at the variation is to estimate the variance-covariance matrix of item parameter estimates. Several alternatives are available for computing the sampling variances of item parameter estimates. The first is to use standard large-sample theory, which holds that the asymptotic variances of $<\hat{a},\hat{b},\hat{c}>$ are given by the inverse of the 3 x 3 Fisher information (I) matrix evaluated at the true parameter values <*a, b,* c> (Lord, 1980; Hambleton, Swaminathan, & Rogers, 1991) defined as,

$$\Sigma_i = \begin{bmatrix} I_{\mathrm{a}} & I_{\mathrm{ab}} & I_{\mathrm{ac}} \\ I_{ab} & I_b & I_{\mathrm{bc}} \\ I_{ac} & I_{\mathrm{bc}} & I_c \end{bmatrix}. \tag{2}$$

The diagonal elements of the matrix represent the information associated with each parameter. The problem, of course, is that the true parameters are unknown. Our best approximation is then to evaluate information at the values of the parameter estimates $<\hat{a},\hat{b},\hat{c}>$ and hope that these are reasonably close to the true values. The estimates were averaged across

nine calibrations to obtain the best estimate for each item. It is, however, true that the item parameter estimates are often constrained to avoid taking on inappropriate values (e.g., negative *a*-parameters or *c*-parameters outside the range [0, 1]). Such constraints are liable to upset asymptotic theory and render the sampling variance approximations less valid.

In the current situation, a second means is available for estimating sampling variation. The items under study were administered on nine separate occasions, and parameter estimates were separately obtained from each administration sample. The observed variation across these estimates is therefore an empirical estimate of the sampling fluctuation of the parameter estimates defined as,

$$\Sigma_i = \begin{bmatrix} \sigma_{\mathrm{a}}^2 & \sigma_{\mathrm{ab}} & \sigma_{\mathrm{ac}} \\ \sigma_{ab} & \sigma_b^2 & \sigma_{\mathrm{bc}} \\ \sigma_{ac} & \sigma_{\mathrm{bc}} & \sigma_c^2 \end{bmatrix}. \tag{3}$$

In theory, and under all of the assumptions of that theory, the empirical and asymptotic estimates of sampling variation should be very similar. However, the empirical variances are only based on nine observations and may not be very stable. Both asymptotic and empirical sampling variance estimates are therefore problematic to some extent. It was therefore decided to repeat the analyses with both.

The last and the most affirming set of analyses was performed to look at the effect of variation in the item parameter estimates on the actual reported scores. The responses of examinees on the anchor items were selected for the nine sets of calibrations on both measures. A typical ability distribution for the examinees during an administration is given in Figure A5 for both Quantitative and Verbal. Each response set was then scored using the set of item parameter estimates obtained on it during calibration process and then using the parameter estimates obtained using each of the other eight sets of responses. Hence 9 sets of scores were obtained from each response set. A grand total of 81 sets of scores were produced from the total of 9 response sets. The scoring was carried out using maximum likelihood estimation methodology (Lord, 1980; Hambleton et al., 1991). RMSE statistics between each set of the baseline theta estimates (or scores obtained using the set of item parameter estimates obtained on the same

response set during calibration process) and the estimates from each of the other eight sets were then computed. The statistic was defined as,

$$RMSE_{ck} = \sqrt{\sum_{j=1}^{n_k} \frac{(\hat{\theta}_{cj} - \hat{\theta}_{kj})^2}{n_k}}, \tag{4}$$

where $\hat{\theta}_{kj}$ is the ability estimate obtained for an examinee *j* on examinee set (or response set) *k* using item parameter estimates obtained by calibrating response set *k*. On the other hand, $\hat{\theta}_{cj}$ is the ability estimate for an examinee *j* on examinee set *k*, using item parameter estimates obtained by calibrating response set *c*.

The ability estimates were then mapped on to the reported score scale and the distributions of differences $(Score_{cj} - Score_{kj})$ for each of the 81 scenarios were plotted. The differences were expressed on the operational or reported score scale where the reported scores for this particular program are expressed in 10-point intervals.

Case Study 2: Adaptive Administration of Items

Estimation Methodology

The second part of this investigation was carried out on a set of adaptively administered operational items from another high stakes admissions test. This particular program uses the item specific prior methodology with a proprietary version of computer software PARSCALE (Muraki & Bock, 1999). This methodology allows unique multivariate normal distributions to be used as prior distributions for the parameters of each item (Swaminathan & Gifford, 1986; Folk & Golub-Smith, 1996). These item specific priors are actually the mean estimates of the (*b, a, c*) parameters as well as the asymptotic variance-covariance matrix specified as (*Intercept, a, c*). These priors are used for the CAT operational items and are different for each item, as they are item specific. On the other hand, global priors are used for the pretest items and are the same for all pretest as well as anchor items. The global prior distributions for the *a*-parameter are approximated by lognormal distribution, *b*-parameter distributions are approximated by normal, and the *c*-parameter prior distributions are approximated by beta distribution. All pretest, anchor, and CAT items are calibrated together for an administration. In this case, pretest or anchor items

are actually embedded in the operational test. This is unlike the previous case, where a pretest or anchor set is offered as a separate section. Since the priors on the CAT items are strong, their values hardly move away from their original values. The CAT items, therefore, set the scale; thus, putting all items on the same scale. Once calibrated, the pretest item parameter estimates are stored in the item bank to be used in subsequent pools, while the operational item parameter estimates are not used further. This methodology has been shown to be effective in utilizing data from operational items that do not have a uniform distribution of ability, since they are administered adaptively.

Data

The data for this investigation came from the Quantitative measure of an adaptively administered high stakes admissions test. Items that had already appeared in operational pools and had been included in several pretest calibrations to hold the scale (with item specific priors on them) were identified. In order to obtain relatively uniform ability distributions, 30 items that were slightly easy, mid-difficulty, or adequately difficult and had sample sizes larger than 500 associated with them were chosen. The item parameter estimates for these items were originally obtained when they were pretested in P & P administrations before the introduction of CAT. The mean and standard deviations for the original *a*, *b*, and *c* parameters are give in Table 3.

Table 3

Mean and Standard Deviations for the a, b, and c Parameters (Original P & P)

	a	*b*	*c*
Mean	1.07	0.23	0.16
Stdev	0.19	0.72	0.05

All chosen items had appeared in several pools and had been included in at least 8 calibrations. The number of calibrations available on these items is given in Table 4.

The ability distributions of examinees who received these items in each calibration were inspected to make sure that the range of examinee abilities for each of these items was not restrictive. For the purpose of this investigation, all calibrations were rerun with the following modification: the item specific priors were removed and global priors were imposed on these

CAT items, thus they were treated like other pretest items. The modified requests for the calibration were resubmitted using ETS-specific software called GENASYS, which uses PARSCALE for calibration. Items were then calibrated in this modified way and new parameter estimates were obtained.

Table 4

Number of Calibrations

No. of items	No. of calibrations
5	8
12	9
7	10
6	11

Analyses

Similar to the previous case study, the item characteristic curves were examined for each item. The weighted RMSEs were then computed between the ICCs for the first calibration, compared with the other calibrations as discussed in the previous study. The first calibration was arbitrarily chosen as the point of comparison.

The next part of the analyses involved looking at the effect of variation in parameter estimates on ability estimation. Unlike the linear case, where a fixed number of calibrations were available on each item, the number of calibrations varied in this case (as shown in Table 5, the number of calibrations on various items varied from 8 to 11). Thus, 20 sets of item parameter estimates were generated for each item by drawing parameters at random from the various calibrations available for that item (except the first calibration). A response set was obtained by generating responses for 1,000 examinees at 11 ability levels corresponding to the ability levels listed in Table 5. These ability levels are obtained on the number-right scale from the reference P & P base form. The number-right score for this particular test ranged from 0 to 60, resulting in 11 ability levels with a 6-point interval. The ability levels when converted on to the theta metric ranged from –3.138 to 2.592.

The item parameter estimates used to generate the response set came from the first calibration and were considered as the baseline estimates. The response set was scored using

baseline parameter estimates from the first calibration, as well as using the 20 other randomly chosen sets of estimates.

Table 5

Ability levels and corresponding weights for Quantitative CAT

Level	Ability	Weight
1	–3.138	0.007
2	–1.970	0.057
3	–1.289	0.107
4	–0.772	0.145
5	–0.337	0.163
6	0.054	0.154
7	0.426	0.135
8	0.800	0.110
9	1.210	0.077
10	1.725	0.039
11	2.592	0.005

The first set of scores was then compared to the other 20 sets of scores. RMSEs were computed between the various sets of ability estimates at each ability level. Since rectangular distribution was simulated, the mean sum-of-squares at various ability levels were weighted in order to compute the overall RMSE. The ability estimates were then converted to scaled or reported scores and the distributions of differences between those scores obtained using various sets of estimates were compared. The differences were expressed on the operational or reported score scale where the reported scores for this particular program are expressed in 1-point intervals.

Next, the scoring analyses were repeated by generating response data using the item bank parameters for these items. As mentioned earlier, these parameters were originally obtained from P & P pretest calibrations. These analyses were expected to reveal more variation in scores due to P & P context effects in addition to positional effects obtained from adaptive administrations. In real calibrations, these estimates are used as priors for the corresponding items; hence, it is important to know whether such context effects influence the parameter estimation. The response set was then scored using the same set of item parameter estimates as well as the remaining 20 sets of estimates.

Results

The results of the analyses on linearly administered items are presented first, followed by the adaptively administered items.

Results for Case Study 1

It should be noted that, for the sake of brevity and clarity, results for Quantitative and Verbal are presented and discussed side-by-side; however, the authors do not intend to compare the two measures. Figure 1 presents the test characteristic curves (TCC) for the set of anchor items for the two measures.

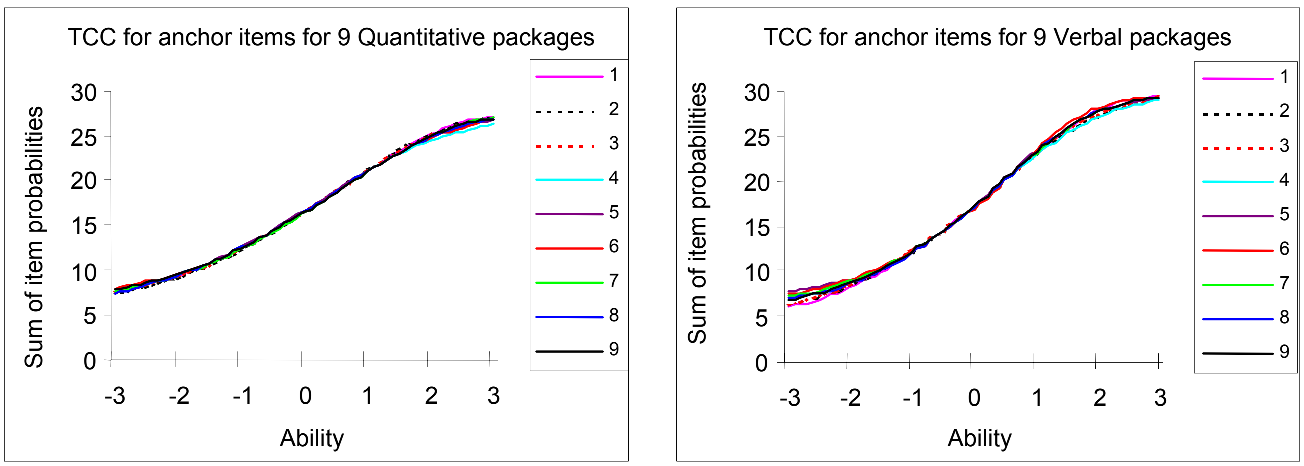

***Figure 1.* TCC for Quantitative and Verbal anchor items over nine calibrations.**

The TCCs for both measures were extremely close under both scenarios. Some variations at the tails of the curve are characteristic of the interaction between the abilities of examinees and difficulty level of the items. Those variations are also shown in the plots of ICCs presented in Figures A1 and A2 in Appendix A. The plots of ICCs for selected Quantitative and Verbal items show that the probabilities of getting an item right did not vary substantially across calibrations, except at the very extreme ends of the scale. The investigation of the general trends did not exhibit any directional change in the estimates. In other words, none of the items exhibited a systematic decrease or increase in the parameter estimates over repeated calibrations.

The RMSEs among ICCs for the two measures are shown in Figure 2. The actual values of the weighted RMSEs are presented in Tables A2 and A3. The RMSEs indicated a small variation between calibrations for both Quantitative and Verbal measures. The differences were slightly higher for Quantitative, especially for some of the items. An item with a very high difficulty level is what appeared to be the most variant in the Quantitative measure. Inspecting the sample sizes and ability distribution for that particular calibration of that item did not suggest any explanation beyond chance-level differences in responding for examinees at extreme ability levels.

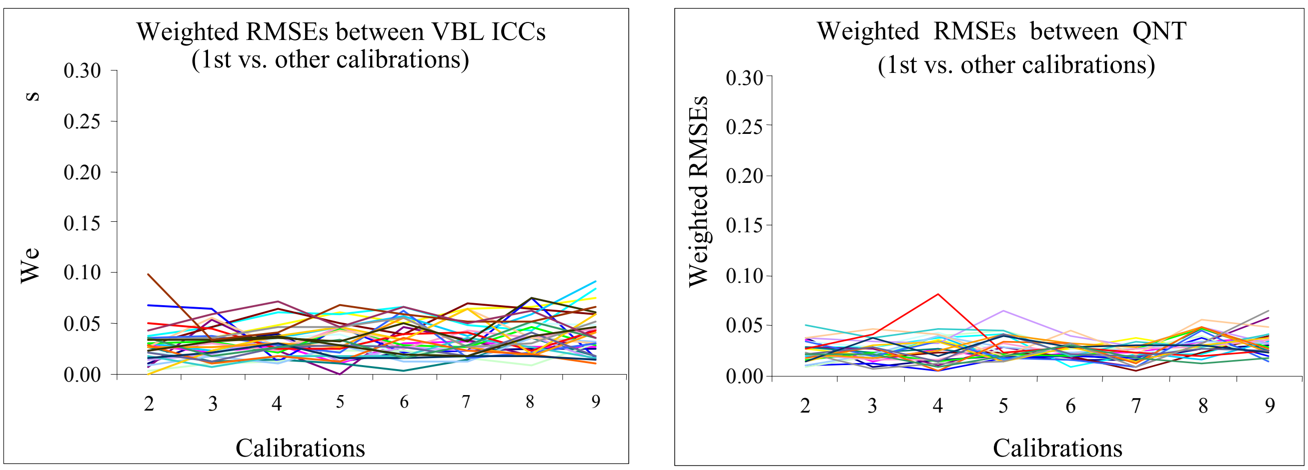

***Figure 2.* Weighted RMSEs between ICCs between first calibration and the others.**

In comparing the model-based vs. empirical variation (see Figures A3 and A4), it was found that the model-based variation was larger than the empirical variation for both Quantitative and Verbal measures for the *b*-parameter. The model-based variance was highly affected by the magnitude of the *b*-parameter: very low *b*-parameters resulted in large values of model-based variance. In the case of the *a*-parameter, model-based variance was larger than the empirical variation for the Quantitative measure while smaller for the Verbal measure. The *a*-parameters for Verbal were, in general, higher in magnitude.

In general, the results did indicate very small model-based and empirical variation in both *a*- and *b*-parameters, except for the model-based variance in *b*-parameter for Quantitative. As these items provide very little information, the extremely low *b*-parameters for some of the Quantitative items caused this variance. In general, these results should be interpreted with caution, as the samples for the analyses were not suitably sized.

The results of scoring using the different sets of parameter estimates are presented in Figures 3 through 6. The results indicated that the RMSEs in ability estimates ranged from 0.13 to 0.33 for Quantitative and 0.13 to 0.34 for Verbal where a response set used in a calibration was scored by item parameters obtained from different response sets (81 cases). Results of two such scenarios are shown in Figure 3, where Quantitative response sets 1 and 9, respectively, are scored using item parameter estimates obtained from each of the other response sets. The figures show that the error in estimates, when scored using different sets of parameter estimates, remained fairly consistent across calibrations. Similar scenarios for Verbal measure are presented in Figure 4.The differences in examinee reported scores, when scored using different sets of item parameter estimates, remained limited to a 0–20 point difference on the reported score scale for majority of the examinees (as mentioned before, the reported scale for this particular program is expressed in 10-point intervals). Of the examinees, 83% to 98% (91% on average) exhibited a 0–20 point score difference for the Quantitative measure. Figure 5 illustrates this result for two typical cases.

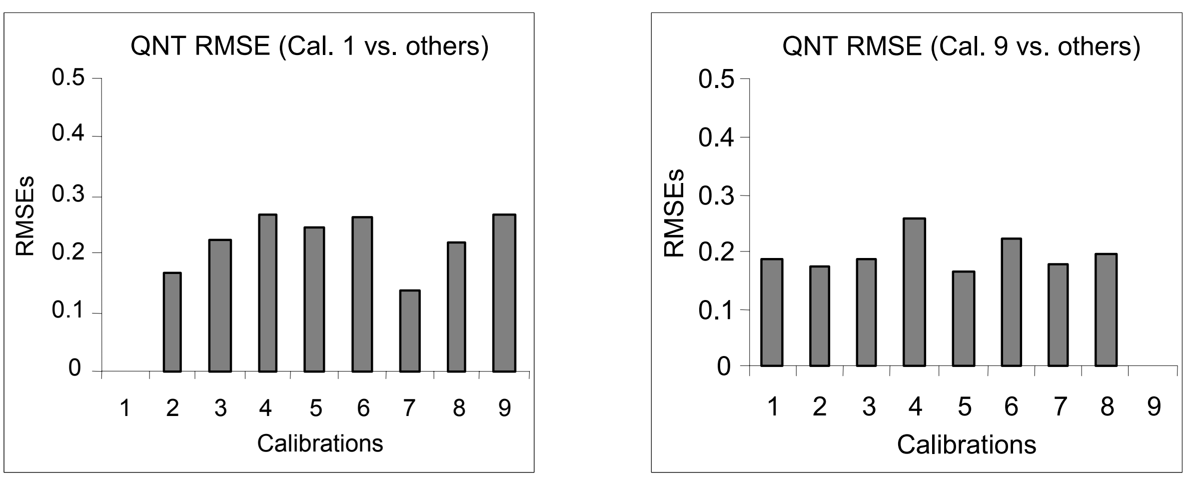

Figure 3. **RMSEs between ability estimates on a response set scored by its own and other sets of item parameter estimates—Quantitative.**

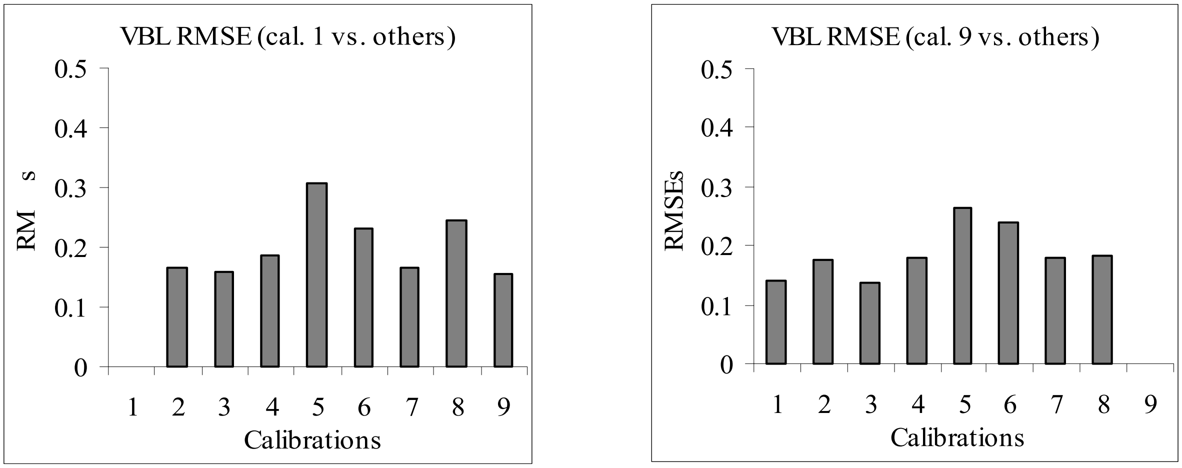

Figure 4. **RMSEs between ability estimates on a response set scored by its own and other sets of item parameter estimates—Verbal.**

The first part of the figure shows response set 2 scored using item parameter estimates obtained on response set 1. The second part shows response set 9 scored using item parameter estimates obtained on response set 5. In the first scenario, 93% of the examinees exhibited a 0–20 point difference in their reported scores, while 90% showed this difference for the second scenario. Similar results for Verbal are shown in Figure 6. The percentages of examinees exhibiting 0–20 point score differences ranged from 87% to 98% (94% on average) for Verbal.

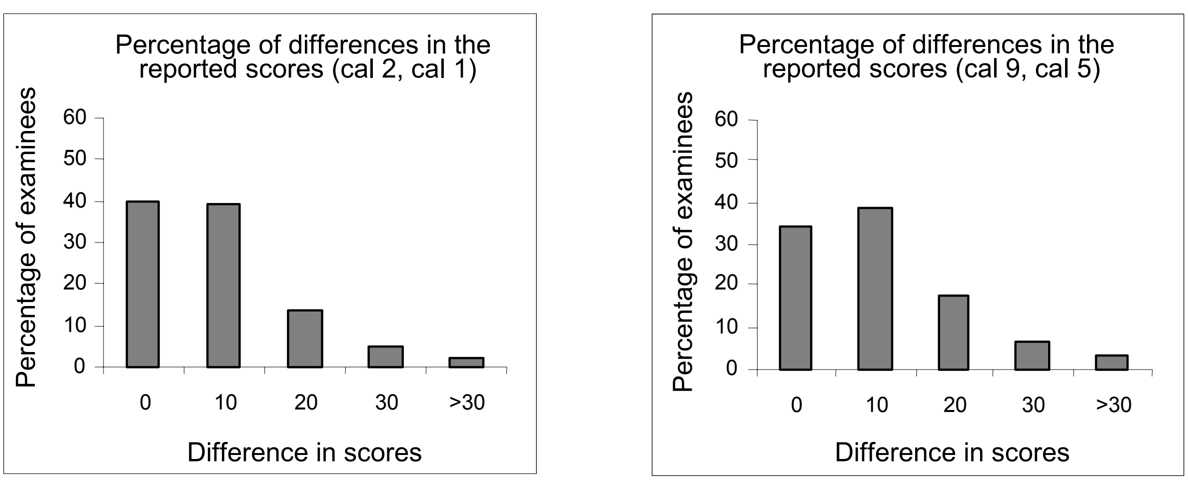

Figure 5. **Frequency distribution of reported score differences for Quantitative.**

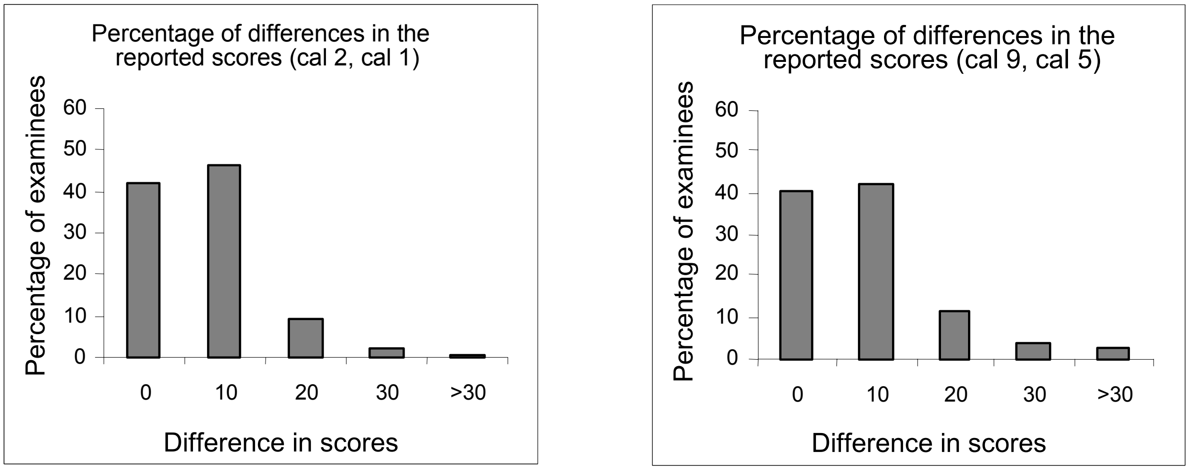

Figure 6. **Frequency distribution of reported score differences for Verbal.**

Since the average Standard Error of Measurement (SEM) ranges from 35 to 45 points for the Quantitative measure and 30 to 40 points for the Verbal measure of this particular program, the results were promising for both measures.

Results for Case Study 2

The ICC plots for selected adaptively administered items are presented in Figure B1 in Appendix B. The weighted RMSEs between ICCs for the adaptive calibrations in comparison with first adaptive calibration are presented in Figure 7. These values are also presented in Tables B1 and B2 for readers' interest. The RMSEs among ICCs for those items revealed remarkably small variation. The values remained in the range of 0.01 and 0.20 for all items for all calibrations.

When compared with scores based on first calibration, the differences in reported scores for the adaptive case, ranged from 0 to 2 points for 90% to 98% (96% on average) of examinees for 20 item parameter sets that were drawn from calibrations. At this point it is worth mentioning again that the reported score scale for this particular program is expressed in 1-point intervals.

The consistency of the RMSEs in the ability estimates across 20 item parameter sets drawn from available calibrations is depicted in Figure 8. When investigated per ability level (Figure 9), a large portion of the error seemed to concentrate in the low ability levels.

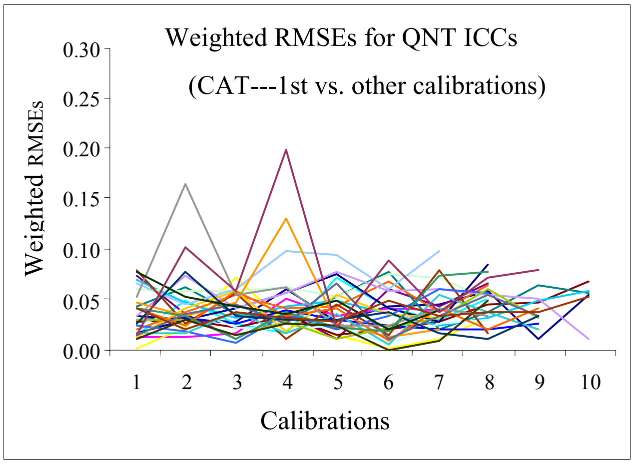

Figure 7. **Weighted RMSEs between CAT ICCs between first calibration and the others.**

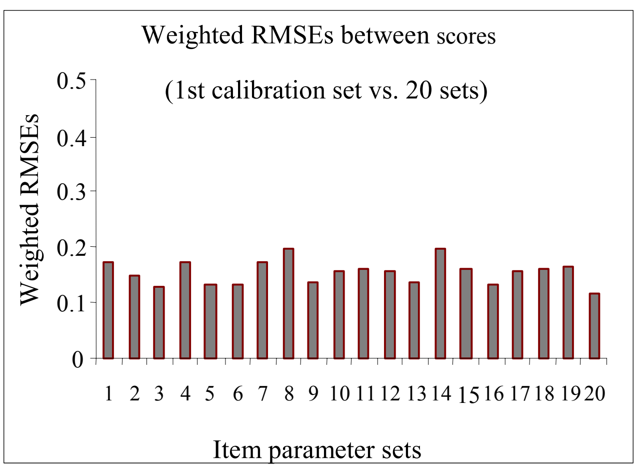

Figure 8. **Weighted RMSEs between ability estimates on a response set scored by its own and another set of item parameter estimates—own set = 1st calibration.**

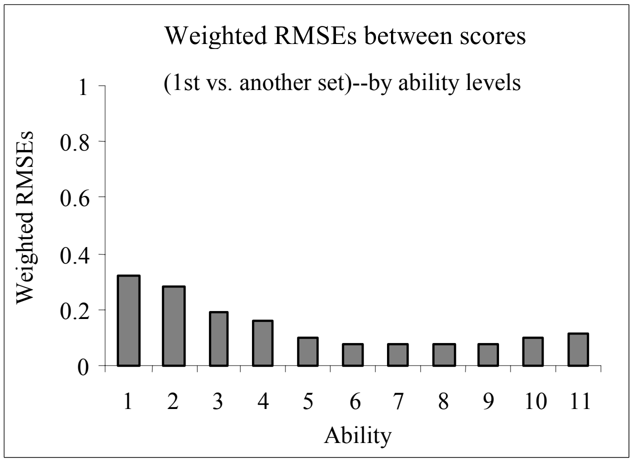

Figure 9. **Weighted RMSEs between ability estimates on a response set scored by its own and another set of item parameters by ability level—own set = 1st calibration.**

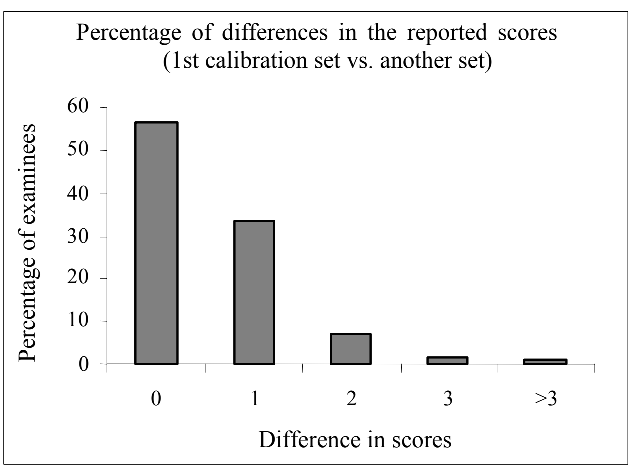

Figure 10. **Frequency distribution of reported score differences—comparison with 1st CBT calibration.**

When P & P calibrated estimates of the items were used in place of the first calibration for comparison between calibrations, the results were quite different. Figure 11 shows the RMSEs between theta estimates obtained on the P & P calibrated sets of parameter estimates and 20 sets of estimates obtained on CBT calibrations. The results indicate an increase of overall RMSEs, when abilities obtained using P & P estimates were used for comparison. While the scenario where comparisons were based on 1st calibration and the overall RMSEs between scores ranged from 0.12 to 0.20, the errors ranged from 0.19 to 0.30 here. The errors in the scores remained significantly small at the middle ability levels, higher for the high ability levels, and highest for the low ability levels, when compared across ability levels. The errors were as high as 0.63 at the lower ability levels.

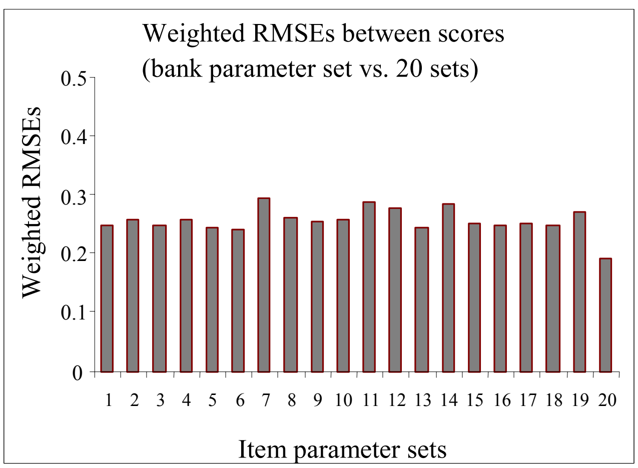

Figure 11. **RMSEs between ability estimates on a response set scored by its own and other sets of item parameter estimates—own set = P & P bank parameters.**

The percentage of examinees that exhibited reported score differences between 0–2 points on the reported score scale ranged from 87% to 94% (91% on average). This percentage was considerably smaller than the previous scenario where most of the cases resulted in more than 93% of the examinees exhibiting a 0–2 point difference. In other words, the percentage of examinees whose scores changed by more than 2 points was significantly large in this case.

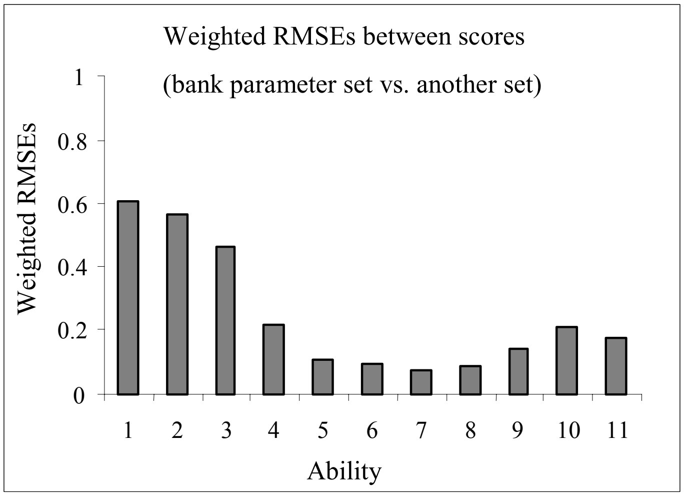

Figure 12. **Weighted RMSEs between ability estimates on a response set scored by its own and another set of item parameters by ability level—own set = P & P bank parameters.**

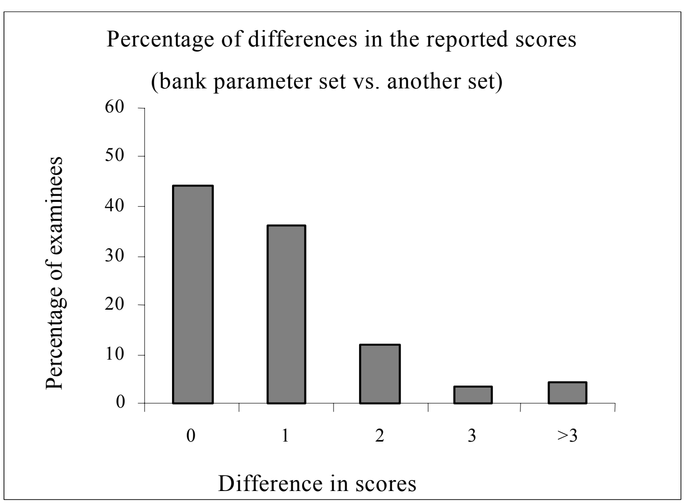

Figure 13. **Frequency distribution of reported score differences—comparison with P & P calibrated parameters.**

The SEM for the Quantitative measure of this program usually ranges from 2.5 to 3.5 points. In the first scenario, 96% of people exhibiting less than or equal to a 2-point difference represented an encouraging result. In other words, 4% of the examinees exhibited a difference of three-or-more points in their reported scores. For the second scenario, where bank parameters were used to score the responses, 9% of the examinees showed a difference of three-or-more points.

Conclusions

The studies discussed in this paper investigated the effect of stability of item parameter estimation in the current CBT calibrations. The results of the study will serve as a baseline for the design work involved in creating models for automated item generation. The concept of having a single model to generate a family of items should be informed by knowing the relative stability of the parameter estimates when calibrated online.

Several conclusions can be drawn from the results of this study. The linearly administered items in a high stakes testing program exhibited remarkably small variation in parameter estimates over repeated calibrations. Although the sample sizes upon which the calibrations were performed varied considerably, the results were not affected. As long as the sample sizes are large enough to calibrate, stable results are produced. Similar findings with adaptively administered items in another high stakes testing program were also found when initial adaptively based item parameter estimates were compared with estimates from repeated use. These findings have implications for research on item modeling because they suggest that the use of item modeling with operational CAT programs will introduce more variation in ability estimation due to item context effects, positional effects, and the small sample sizes obtained for some items. It will be important to quantify and account for these sources of variation as this research progresses.

The results of this study also indicate that context effects played a more significant role in adaptive item parameters when the comparisons were made to the parameters that were obtained from P & P testing. Even though PARSCALE was used to calibrate both sets of items, however, P & P items went through concurrent calibrations as opposed to item-specific prior methodology used for adaptive items; this fact may also have caused some variation. This suggests that the parameter estimates obtained on P & P administrations should be replaced,

whenever feasible, with the CBT calibrated parameters. The approach employed for this paper (i.e., freeing the item specific priors that constrain item parameter estimates for selected operational items during the process of pretest item calibration) is one possible alternative for this kind of updating. However, further research would be necessary to determine if this approach would be feasible in the context of an ongoing, operational CAT program.

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Appendix A

Table A1

Sample Sizes for Each Calibration

Calibration	Total sample	Act. sample size per anchor item	# of perfect scores	Final sample
QNT				
1	6,656	1,299	8	1,291
2	10,178	1,420	15	1,405
3	16,311	1,182	7	1,175
4	20,018	1,115	11	1,104
5	6,038	833	8	825
6	17,949	1,432	6	1,426
7	19,863	2,323	18	2,305
8	16,493	858	14	844
9	20,422	636	9	627
VBL				
1	13,632	2,287	3	2,284
2	8,774	1,066	2	1,064
3	13,329	992	0	992
4	14,697	1,118	4	1,114
5	15,151	1,047	2	1,045
6	11,026	876	3	873
7	2,130	1,569	2	1,567
8	5,869	834	4	830
9	24,945	939	2	937

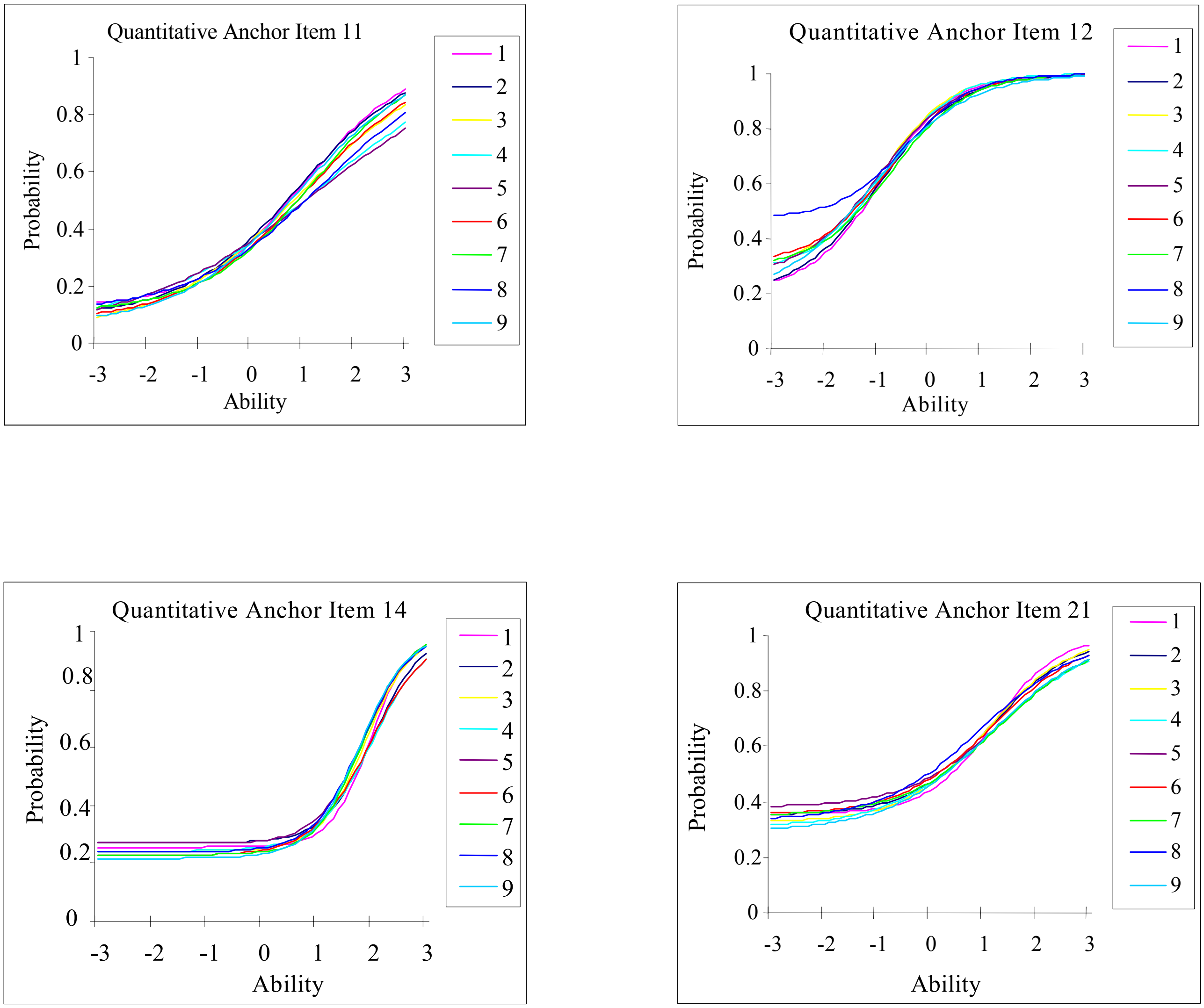


Figure A1. **ICCs for four Quantitative items over nine calibrations.**

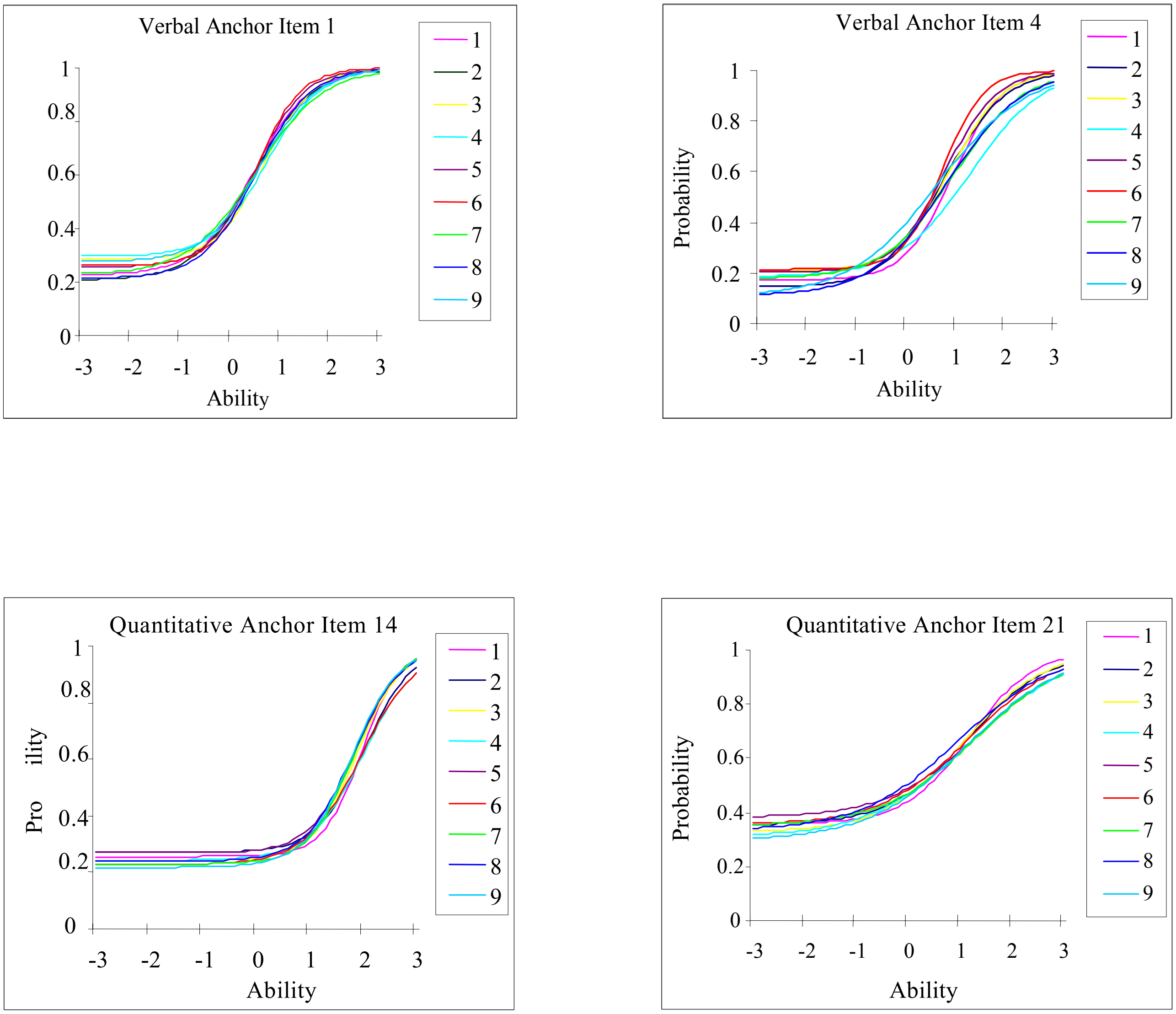


Figure A2. **ICCs for four Verbal items over nine calibrations.**

Table A2

Weighted RMSEs in ICCs for Quantitative Measure

	2	3	4	5	6	7	8	9
1	0.04	0.01	0.02	0.02	0.02	0.02	0.03	0.03
2	0.03	0.01	0.03	0.02	0.02	0.02	0.03	0.04
3	0.03	0.02	0.04	0.02	0.03	0.04	0.03	0.03
4	0.03	0.03	0.04	0.04	0.01	0.02	0.02	0.04
5	0.03	0.03	0.01	0.03	0.02	0.01	0.03	0.06
6	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.04
7	0.03	0.02	0.03	0.02	0.02	0.01	0.05	0.02
8	0.01	0.01	0.00	0.02	0.02	0.02	0.04	0.02
9	0.02	0.02	0.04	0.01	0.03	0.02	0.02	0.03
10	0.03	0.03	0.04	0.02	0.03	0.02	0.03	0.04
11	0.01	0.02	0.04	0.04	0.03	0.03	0.04	0.02
12	0.02	0.02	0.01	0.02	0.02	0.03	0.05	0.03
13	0.01	0.02	0.04	0.03	0.02	0.03	0.03	0.03
14	0.02	0.02	0.01	0.03	0.02	0.03	0.03	0.04
15	0.04	0.03	0.03	0.06	0.04	0.03	0.05	0.03
16	0.04	0.05	0.04	0.02	0.04	0.02	0.06	0.05
17	0.03	0.02	0.03	0.02	0.02	0.01	0.04	0.01
18	0.05	0.04	0.05	0.04	0.02	0.03	0.03	0.04
19	0.03	0.04	0.08	0.02	0.03	0.02	0.02	0.02
20	0.01	0.03	0.03	0.01	0.03	0.01	0.03	0.04
21	0.02	0.02	0.02	0.04	0.03	0.03	0.05	0.02
22	0.02	0.03	0.00	0.03	0.03	0.01	0.05	0.03
23	0.02	0.03	0.01	0.04	0.02	0.02	0.03	0.02
24	0.02	0.01	0.01	0.01	0.03	0.01	0.03	0.07
25	0.01	0.04	0.02	0.04	0.03	0.03	0.03	0.02
26	0.02	0.02	0.01	0.02	0.03	0.02	0.01	0.02
27	0.02	0.04	0.04	0.05	0.04	0.01	0.03	0.02
28	0.02	0.03	0.04	0.04	0.03	0.03	0.02	0.02
Average	0.02	0.02	0.03	0.03	0.03	0.02	0.03	0.03

Table A3

Weighted RMSEs in ICCs for Verbal Measure

	2	3	4	5	6	7	8	9
1	0.01	0.03	0.04	0.01	0.02	0.02	0.02	0.02
2	0.03	0.02	0.03	0.01	0.03	0.03	0.03	0.03
3	0.03	0.03	0.05	0.06	0.05	0.06	0.07	0.08
4	0.04	0.05	0.06	0.06	0.07	0.05	0.04	0.08
5	0.01	0.05	0.02	0.00	0.04	0.03	0.02	0.04
6	0.03	0.05	0.06	0.05	0.04	0.07	0.06	0.06
7	0.03	0.01	0.01	0.01	0.00	0.01	0.03	0.02
8	0.07	0.07	0.01	0.05	0.02	0.02	0.07	0.01
9	0.03	0.02	0.04	0.05	0.06	0.04	0.06	0.09
10	0.02	0.02	0.04	0.04	0.02	0.03	0.05	0.05
11	0.00	0.01	0.01	0.02	0.02	0.01	0.01	0.03
12	0.03	0.03	0.02	0.04	0.03	0.03	0.05	0.03
13	0.01	0.02	0.01	0.03	0.01	0.01	0.04	0.03
14	0.02	0.03	0.02	0.05	0.02	0.04	0.04	0.04
15	0.01	0.02	0.04	0.01	0.03	0.02	0.04	0.01
16	0.03	0.06	0.03	0.04	0.03	0.07	0.04	0.03
17	0.04	0.04	0.03	0.02	0.06	0.02	0.02	0.03
18	0.02	0.01	0.02	0.02	0.02	0.03	0.03	0.02
19	0.05	0.04	0.02	0.02	0.04	0.04	0.02	0.04
20	0.00	0.02	0.04	0.05	0.03	0.06	0.02	0.06
21	0.03	0.03	0.03	0.03	0.05	0.03	0.02	0.04
22	0.03	0.01	0.02	0.01	0.04	0.02	0.02	0.01
23	0.02	0.01	0.03	0.03	0.03	0.02	0.04	0.02
24	0.03	0.04	0.05	0.05	0.06	0.03	0.03	0.05
25	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.01
26	0.03	0.02	0.03	0.03	0.02	0.03	0.05	0.04
27	0.03	0.03	0.04	0.03	0.02	0.02	0.04	0.04
28	0.02	0.03	0.04	0.03	0.05	0.03	0.08	0.06
29	0.04	0.06	0.07	0.05	0.07	0.05	0.06	0.04
30	0.10	0.03	0.04	0.07	0.06	0.05	0.05	0.07
Average	0.03	0.03	0.03	0.03	0.04	0.03	0.04	0.04

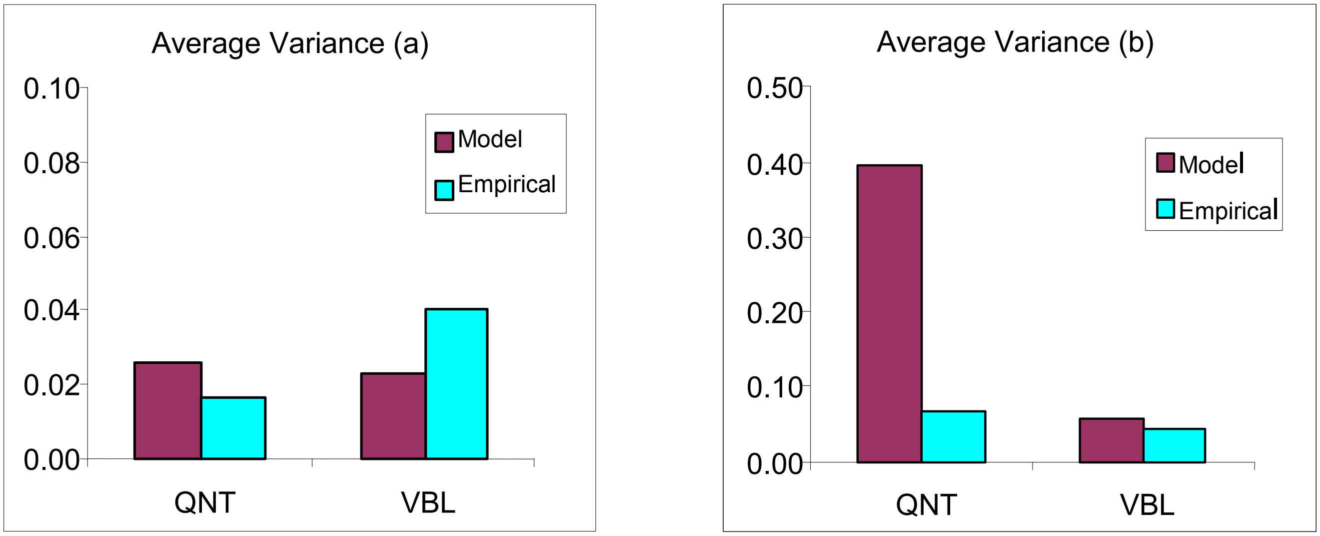


Figure A3. **Model-based vs. empirical average variance for a- and b-parameters.**

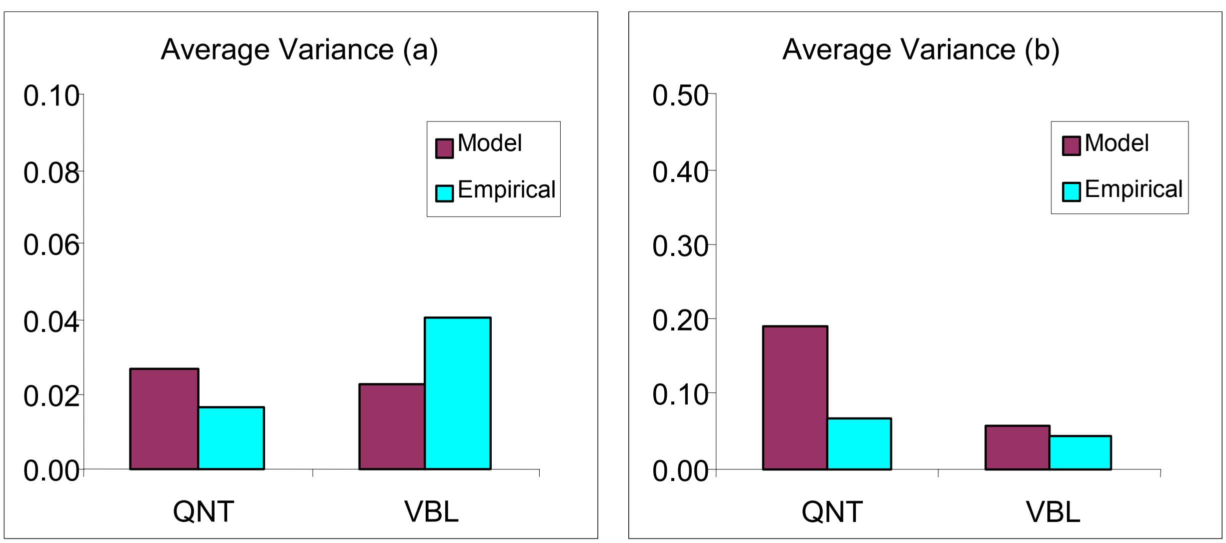
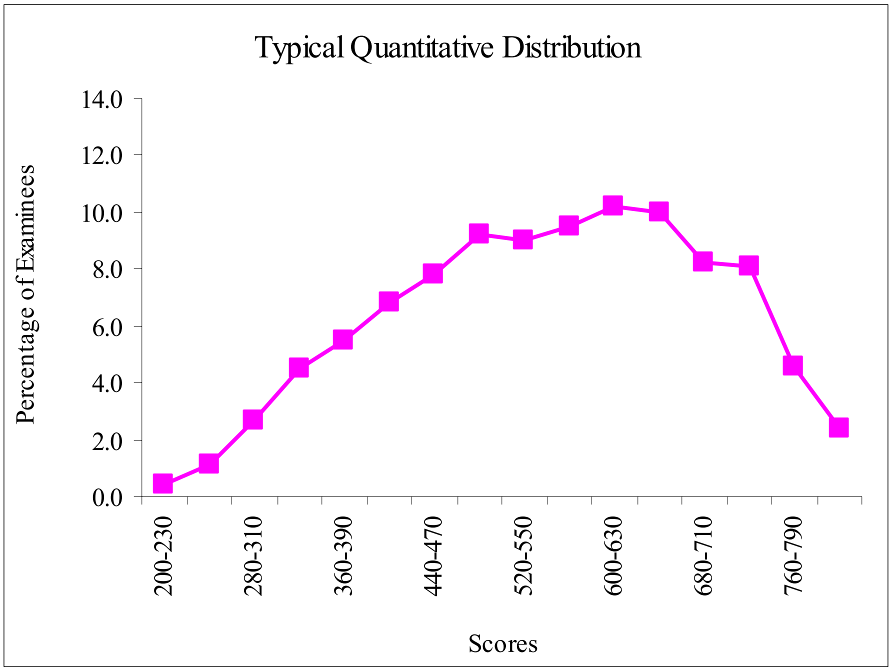


Figure A4. **Model-based vs. empirical average variance for *a*- and *b*-parameters after deleting two very easy Quantitative items.**



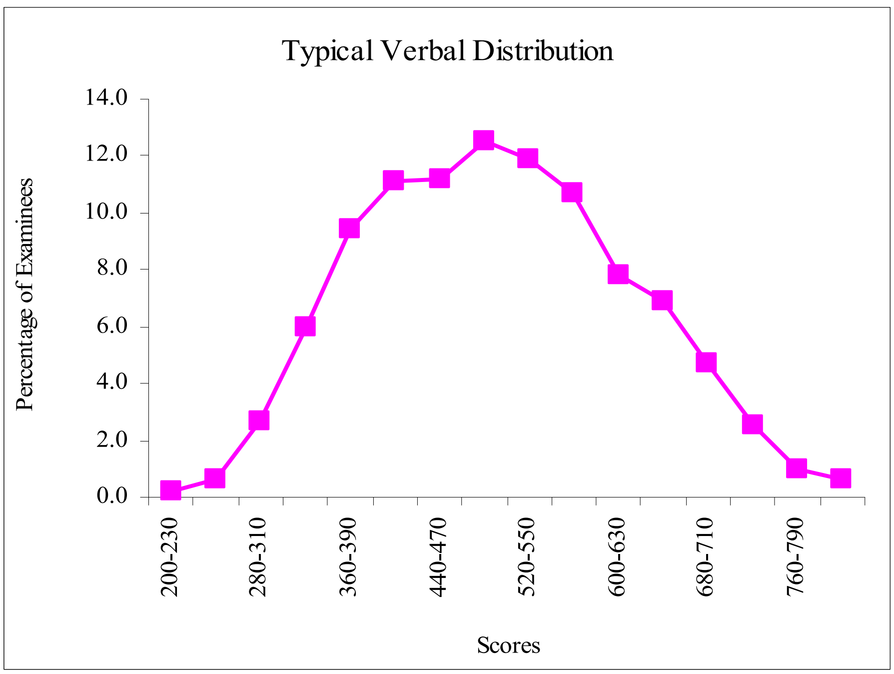


Figure A5. **Typical ability distributions for Quantitative and Verbal measures.**

Appendix B

Results for Adaptively Administered Items

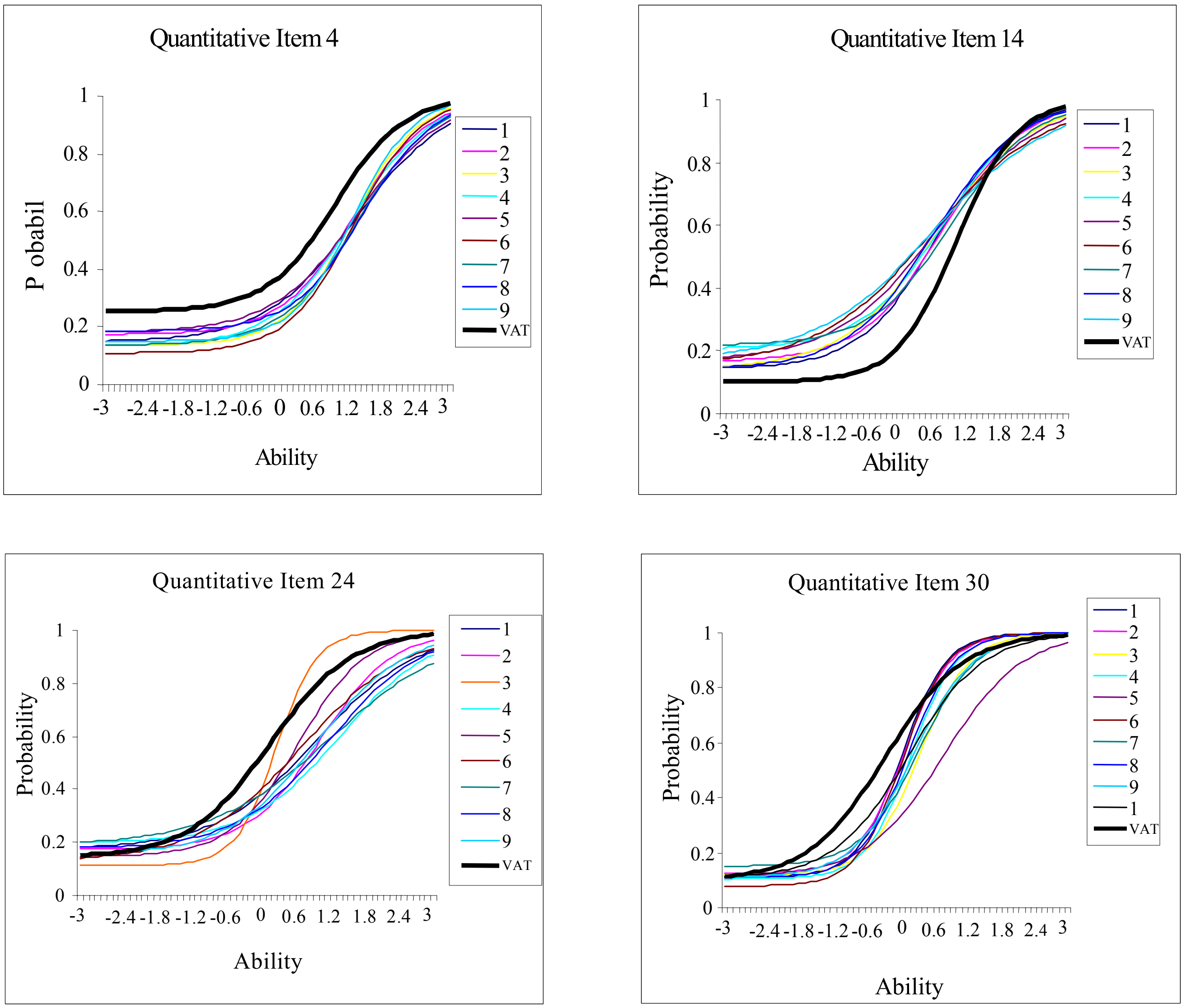


Figure B1. **ICCs for four Quantitative CAT items.**

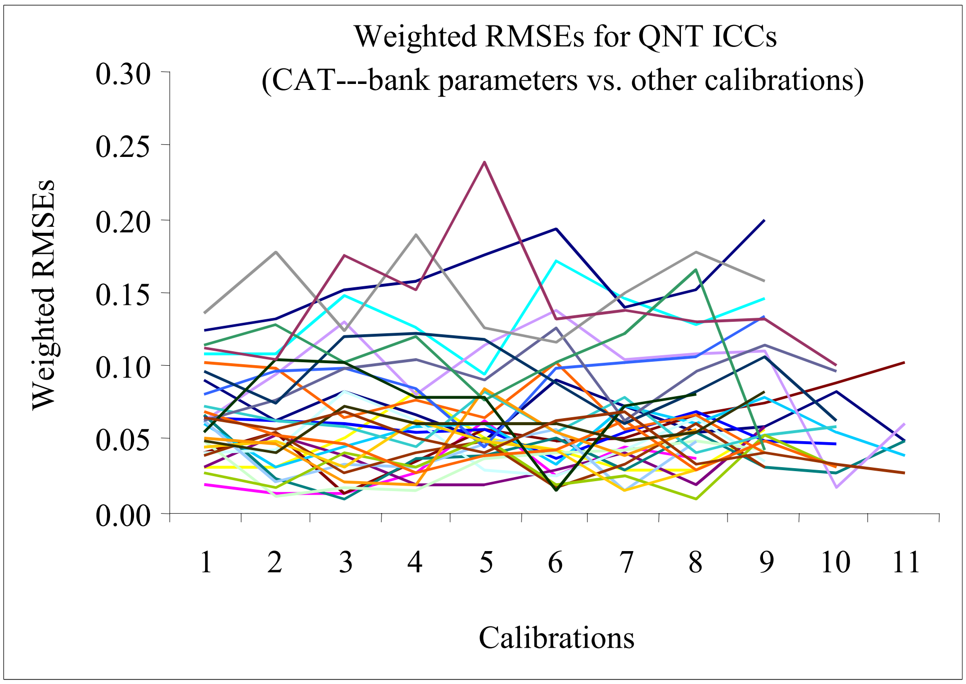


Figure B2. **Weighted RMSEs in ICCs for CAT items on Quantitative measure.**

Table B1

Weighted RMSEs in ICCs for CAT Items on Quantitative Measure

	2	3	4	5	6	7	8	9[a]	10[a]	11[a]
1	0.03	0.03	0.03	0.06	0.03	0.04	0.05	0.06	0.01	0.05
2	0.01	0.01	0.02	0.05	0.03	0.04	0.04			
3	0.00	0.02	0.07	0.03	0.01	0.00	0.01	0.03	0.03	
4	0.01	0.05	0.03	0.02	0.07	0.04	0.03	0.05	0.11	
5	0.07	0.04	0.01	0.04	0.01	0.06	0.04	0.07	0.03	
6	0.03	0.03	0.02	0.03	0.01	0.02	0.03	0.04	0.05	0.07
7	0.04	0.06	0.04	0.03	0.05	0.08	0.02	0.04	0.06	0.06
8	0.03	0.03	0.03	0.04	0.03	0.02	0.02	0.02	0.03	
9	0.07	0.05	0.03	0.04	0.05	0.03	0.02	0.03	0.05	0.06
10	0.02	0.04	0.02	0.02	0.02	0.00	0.02	0.03		
11	0.06	0.06	0.06	0.06	0.05	0.07	0.07	0.05	0.05	
12	0.02	0.04	0.05	0.04	0.04	0.07	0.04	0.06	0.10	
13	0.07	0.04	0.06	0.10	0.09	0.06	0.10			
14	0.02	0.03	0.04	0.06	0.08	0.04	0.03	0.08		
15	0.03	0.07	0.04	0.06	0.08	0.06	0.06	0.05	0.05	0.01
16	0.08	0.03	0.06	0.01	0.04	0.01	0.08	0.02		
17	0.02	0.02	0.01	0.04	0.02	0.03	0.06	0.06		
18	0.02	0.02	0.04	0.02	0.04	0.00	0.05	0.04	0.02	
19	0.03	0.03	0.02	0.03	0.01	0.02	0.03	0.06	0.03	
20	0.01	0.04	0.06	0.02	0.05	0.04	0.03			
21	0.05	0.03	0.05	0.13	0.03	0.01	0.02			
22	0.04	0.02	0.06	0.03	0.04	0.01	0.04	0.02	0.04	
23	0.01	0.03	0.04	0.03	0.07	0.01	0.03	0.05	0.03	
24	0.05	0.17	0.05	0.06	0.02	0.02	0.04	0.03		
25	0.03	0.08	0.03	0.03	0.03	0.04	0.02	0.01	0.03	
26	0.04	0.03	0.01	0.04	0.02	0.02	0.07	0.08		
27	0.08	0.05	0.04	0.03	0.05	0.02	0.04			
28	0.01	0.03	0.01	0.03	0.02	0.00	0.01	0.05		
29	0.04	0.02	0.04	0.03	0.03	0.05	0.03	0.04	0.04	0.05
30	0.01	0.10	0.06	0.20	0.02	0.09	0.04	0.07	0.08	

[a] Some cells are empty as the number of calibrations varied from 8 to 11 for different items.

Table B2

Weighted RMSEs in ICCs for CAT Items on Quantitative Measure (P & P or Bank Parameter Estimates)

	1	2	3	4	5	6	7	8	9 [a]	10 [a]	11 [a]
1	0.09	0.06	0.08	0.07	0.05	0.09	0.07	0.06	0.06	0.08	0.05
2	0.02	0.01	0.01	0.03	0.06	0.02	0.05	0.04			
3	0.03	0.03	0.05	0.08	0.05	0.04	0.03	0.03	0.06		
4	0.11	0.11	0.15	0.13	0.10	0.17	0.15	0.13	0.15		
5	0.03	0.05	0.04	0.02	0.02	0.03	0.04	0.02	0.06		
6	0.04	0.06	0.01	0.03	0.06	0.05	0.05	0.07	0.08	0.09	0.10
7	0.07	0.02	0.01	0.04	0.04	0.05	0.03	0.06	0.03	0.03	0.05
8	0.06	0.06	0.06	0.06	0.06	0.04	0.06	0.07	0.05	0.05	
9	0.06	0.03	0.05	0.06	0.06	0.03	0.07	0.06	0.08	0.05	0.04
10	0.04	0.04	0.08	0.06	0.03	0.03	0.04	0.07	0.03		
11	0.05	0.01	0.02	0.01	0.04	0.04	0.04	0.05	0.04		
12	0.10	0.10	0.07	0.08	0.06	0.10	0.06	0.07	0.04		
13	0.06	0.02	0.03	0.03	0.05	0.06	0.02	0.05			
14	0.12	0.13	0.15	0.16	0.17	0.19	0.14	0.15	0.20		
15	0.06	0.09	0.13	0.08	0.11	0.14	0.10	0.11	0.11	0.02	0.06
16	0.04	0.06	0.03	0.04	0.05	0.02	0.03	0.06	0.03		
17	0.08	0.10	0.10	0.09	0.05	0.10	0.10	0.11	0.13		
18	0.07	0.06	0.06	0.05	0.08	0.06	0.08	0.04	0.05	0.06	
19	0.03	0.02	0.04	0.03	0.05	0.02	0.02	0.01	0.05	0.03	
20	0.05	0.05	0.03	0.06	0.05	0.04	0.01	0.03			
21	0.05	0.05	0.02	0.02	0.09	0.05	0.04	0.06			
22	0.07	0.05	0.05	0.03	0.04	0.04	0.06	0.03	0.05	0.03	
23	0.06	0.08	0.10	0.10	0.09	0.13	0.06	0.10	0.11	0.10	
24	0.14	0.18	0.12	0.19	0.13	0.12	0.15	0.18	0.16		
25	0.10	0.07	0.12	0.12	0.12	0.09	0.06	0.08	0.11	0.06	
26	0.11	0.13	0.10	0.12	0.08	0.10	0.12	0.16	0.04		
27	0.06	0.11	0.10	0.08	0.08	0.02	0.07	0.08			
28	0.05	0.04	0.07	0.06	0.06	0.06	0.05	0.06	0.08		
29	0.07	0.06	0.07	0.05	0.04	0.06	0.07	0.03	0.04	0.03	0.03
30	0.11	0.11	0.18	0.15	0.24	0.13	0.14	0.13	0.13	0.10	

[a] Some cells are empty as the number of calibrations varied from 8 to 11 for different items.